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CONCENTRATED FORCE IN A TRANSVERSALLY-ISOTROPIC HALF-SPACE AND IN A COMPOSITE SPACE

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The problem of the effect of a concentrated force in an isotropic (orthotropic) space has been examined in [1–3].

The problem is investigated below by the method of complex Smirnov-Sobolev solutions, generalized to a system of differential equations.

The results obtained are of elementary nature just for a transversally isotropic solid.

1. Complex solutions of the equilibrium equations. If the potentials φ, ψ, χ are introduced by assuming

$$u = \frac{\partial\varphi}{\partial x} + \frac{\partial\psi}{\partial y}, \quad v = \frac{\partial\varphi}{\partial y} - \frac{\partial\psi}{\partial x}, \quad w = \frac{\partial\chi}{\partial z} \tag{1.1}$$

then the equilibrium equations of a transversally isotropic body under the condition that the z-axis is along the axis of elastic symmetry become

$$\frac{\partial L_1}{\partial x} + \frac{\partial Q}{\partial y} = 0, \quad \frac{\partial L_1}{\partial y} - \frac{\partial Q}{\partial x} = 0, \quad \frac{\partial L_2}{\partial z} = 0 \tag{1.2}$$

$$L_1 = A\,\Delta\varphi + Ld^2\varphi / dz^2 + (L+F)d^2\chi / dz^2$$

$$L_2 = (L+F)\Delta\varphi + L\Delta\chi + Cd^2\chi / dz^2 \tag{1.3}$$

$$Q = N\Delta\psi + Ld^2\psi / dz^2, \qquad \Delta = d^2/dx^2 + d^2/dy^2$$

Here A, L, F, N, C are elastic constants [4]. Let us construct the solution of the system (1.2) in the form

$$\varphi = \operatorname{Re} \varphi^\circ(\theta), \qquad \psi = \operatorname{Re}\psi^\circ(\theta), \qquad \chi = \operatorname{Re}\chi^\circ(\theta) \tag{1.4}$$

The variable θ is defined by the relationship

$$\delta = \alpha\xi + \beta\eta + \nu\zeta + f(\theta) = 0, \qquad \alpha = \cos\theta, \ \beta = \sin\theta$$
$$\xi = x - x_0, \qquad \eta = y - y_0, \qquad \zeta = z - z_0 \tag{1.5}$$

where the function $f(\theta)$ is arbitrary.

Complying with (1.2), and utilizing the differentiation formulas [5]

$$\frac{\partial^3\psi}{\partial x^2\,\partial y} = -\operatorname{Re}\frac{1}{\delta'}\frac{\partial}{\partial\theta}\left[\frac{1}{\delta'}\frac{\partial}{\partial\theta}\left(\frac{\alpha^2\beta\varphi^{\circ\prime}}{\delta'}\right)\right],$$
$$\delta' = -\beta\xi + \alpha\eta + \nu'\zeta + f'(\theta) \tag{1.6}$$

we obtain

$$(A + \nu^2 L)\varphi^{\circ\prime} + (L + F)\nu^2\chi^{\circ\prime} = 0, \qquad (L + F)\,\varphi^{\circ\prime} + (L + \nu^2 C)\chi^{\circ\prime} = 0 \tag{1.7}$$

$$(N + \nu^2 L)\psi^{\circ\prime} = 0 \tag{1.8}$$

From (1.7) we deduce

$$\begin{vmatrix} A + \nu^2 L & (L + F)\,\nu^2 \\ L + F & L + \nu^2 C \end{vmatrix} = 0 \tag{1.9}$$

i.e. the function $\nu(\theta)$ is constant in the anisotropy case under consideration, and equals the roots $\pm i\nu_1, \pm i\nu_2$ of (1.9). For simplicity, we consider the ν_k in the latter to be real positive numbers.

A particular solution of (1.7)

$$\varphi_k^{\circ\prime}(\theta_k) = (L - \nu_k^2 C)\,\omega_k(\theta_k), \qquad \chi^{\circ\prime}(\theta_k) = -(L + F)\,\omega_k(\theta_k) \tag{1.10}$$

corresponds to each root ν_k, where the function ω_k is arbitrary.

From (1.8) we deduce $\nu = i\nu_3$, $\nu_3 = \sqrt{N/L}$, $\psi^{\circ\prime}$ is arbitrary.

The variable θ_k $(k = 1, 2, 3)$ is defined by the relationship

$$\delta_k = \alpha_k\xi + \beta_k\eta \pm i\nu_k\zeta + f_k(\theta_k) = 0 \tag{1.11}$$

According to (1.1)

$$u = -\operatorname{Re}\left[\sum_{k=1}^{2}(L - \nu_k^2 C)\frac{\omega_k}{\delta_k'}\alpha_k + \frac{\beta_3\omega_3}{\delta_3'}\right]$$

$$v = -\operatorname{Re}\left[\sum_{k=1}^{2}(L - \nu_k^2 C)\frac{\omega_k}{\delta_k'}\beta_k - \frac{\alpha_3\omega_3}{\delta_3'}\right] \tag{1.12}$$

$$w = (L + F)\operatorname{Re}\sum_{k=1}^{2}\frac{i\nu_k\omega_k}{\delta_k'}, \qquad \omega_3 = \psi^{\circ\prime}$$

$$\delta_k' = -\beta_k\xi + \alpha_k\eta + f_k'(\theta_k) \qquad (k = 1, 2, 3)$$

Formulas (1.12) contain the arbitrary functions ω_k, f_k and define a class of complex solutions of the equilibrium equations of the considered anisotropic medium. An analogous class of solutions can be constructed for the equilibrium equations of a medium with a general kind of anisotropy. The selection of the potentials (1.1) does not limit the generality of the solutions in the class (1.12) since a mutually one-to-one correspondence can be established between them and the "complete" system of potentials governing the longitudinal and transverse displacements.

Let us examine particular cases.

a) Plane solutions. Let us put $\theta_k = \theta_0 = \text{const}$, $f_k \equiv -\theta_k$, then

$$\alpha_k = \cos\theta_0, \qquad \beta_k = \sin\theta_0, \qquad \delta_k' = -1$$
$$\theta_k = \xi\cos\theta_0 + \eta\sin\theta_0 \pm i\nu_k\zeta \tag{1.13}$$

The solution (1.12) is written as

$$u = \text{Re}\ \{[(L - \nu_1^2C)\,\omega_1 + (L - \nu_2^2C)\omega_2]\cos\theta_0 - \omega_3\sin\theta_0\}$$
$$v = \text{Re}\ \{[(L - \nu_1^2C)\omega_1 + (L - \nu_2^2C)\omega_2]\sin\ \theta_0 + \omega_3\cos\theta_0\} \tag{1.14}$$
$$w = \text{Re}\ [-(L + F)i\ (\nu_1\omega_1 + \nu_1\omega_2)]$$

In addition to the arbitrary analytic functions ω_k, it contains the arbitrary parameter θ_0, the governing solution of the plane problem in a plane passing through the z-axis and making the angle θ_0 with the xz-plane. Integrating (1.14) with respect to θ_0 between 0 and 2π, we obtain a new solution of the equilibrium equations (1.2). The development in this direction has been expounded in [6].

b) Homogeneous solutions. We obtain these by putting $f_k \equiv 0$ in (1.11). In this case we have

$$\alpha_k = \rho^{-2}\,(R_k\eta - i\nu_k\zeta\xi), \qquad \beta_k = -\rho^{-2}\,(R_k\xi + i\nu_k\zeta\eta)$$
$$\delta_k' = -\beta_k\xi + \alpha_k\eta = R_k = (\rho^2 + \nu_k^2\zeta^2)^{1/2}, \qquad \rho^2 = \xi^2 + \eta^2 \tag{1.15}$$

Just as in [7], it can be shown that solutions of the equilibrium equations of an anisotropic medium corresponding to the effect of a concentrated force at a point of infinite space or at a point of a half-space boundary are contained in this class.

2. Concentrated force in infinite space. Let us place the origin at the point where a concentrated force of intensity P acts in the direction of the z-axis.

We put $\omega_k = iD_k$ in the solution (1.12), where D_k are real constants and $\omega_3 = 0$ (no torsion). Then, taking account of (1.15), we obtain after separating out the real part and demanding boundedness of the radial displcement at $\rho = 0$

$$u_\rho = -\sum_{k=1}^{2} \frac{(L - \nu_k^2C)\,D_k}{R_kR_k^*}, \qquad w = -(L + F)\sum_{k=1}^{2} \frac{\nu_kD_k}{R_k}$$
$$(R_k^* = R_k + \nu_kz) \tag{2.1}$$

The following condition is hence imposed on the D_k

$$(L - \nu_1^2C)D_1 + (L + \nu_2^2C)D_2 = 0 \tag{2.2}$$

We derive another relationship from the requirement of equivalence between the loading due to stresses distributed over a small sphere described around the origin and the applied concentrated force P. We hence obtain

$$(F + \nu_1^2C)D_1 + (F + \nu_2^2C)D_2 = -P\ /\ 4\pi L \tag{2.3}$$

Substituting the value of D_k into (2.1), we finally write

$$u_\rho = \frac{E_0(L + F)}{\nu_2 - \nu_1}\left[\frac{1}{R_1R_1^*} - \frac{1}{R_2R_2^*}\right]$$
$$w = -\frac{E_0}{\nu_2 - \nu_1}\left[\frac{\nu_1(L - \nu_2^2C)}{R_1} - \frac{\nu_2(L - \nu_1^2C)}{R_2}\right] \tag{2.4}$$
$$E_0 = P\ [4\pi LC\ /\ \nu_1 + \nu_2)]^{-1}$$

Putting $C = A = \lambda + 2\mu$, $L = \mu$, $F = \lambda$, where λ, μ are Lamé coefficients, we obtain the appropriate result for an isotropic medium after resolving the indeterminacy.

3. Concentrated force at a point of a half-space. If a concentrated force P is applied at a point $M_0\,(x_0,\ y_0,\ z_0)$ of a half-space, then we write (2.4) as

$$u = u_1 + u_2, \qquad v = v_1 + v_2, \qquad w = w_1 + w_2 \tag{3.1}$$

where p is the solution connected with the variable θ_p defined by the relationships

$$\delta_p = \xi\alpha_p + \eta\beta_p + i\nu_p\,(z - z_0) = 0$$
$$\alpha_p^2 + \beta_p^2 = 1 \qquad (p = 1,\ 2) \tag{3.2}$$

Let us put the solution with subscript d in correspondence with the pq-solution u_{pq}°, v_{pq}°, w_{pq}° connected with the variable θ_{pq} defined by the relationship

$$\delta_{pq} = \xi\alpha_{pq} + \eta\beta_{pq} - i\nu_p z_0 - i\nu_q z = 0$$
$$\alpha_{pq}^2 + \beta_{pq}^2 = 1, \qquad \delta_{pq}' = R_{pq} = [\rho^2 + (\nu_p z_0 + \nu_q z)^2]^{1/2} \tag{3.3}$$

This latter is defined uniquely by the demand of coincidence of all the variables on the half-space boundary $z = 0$.

We find the solution with subscript pq from the condition that the stresses σ_{zp}^*, ν_{zxp}^*, τ_{zyp}^* corresponding to the particular solution

$$u_p^* = u_p + u_{p1}^\circ + u_{p2}^\circ, \qquad v_p^* = v_p + v_{p1}^\circ + v_{p2}^\circ$$
$$w_p^* = w_p + w_{p1}^\circ + w_{p2}^\circ \qquad (p = 1,\ 2) \tag{3.4}$$

vanish at $z = 0$.

Let us introduce potentials by taking account of the absence of torsion

$$u_p^* = \frac{\partial\varphi_p^*}{\partial x}, \qquad v_p^* = \frac{\partial\varphi_p^*}{\partial y}, \qquad w_p^* = \frac{\partial\chi_p^*}{\partial z} \tag{3.5}$$

Utilizing (1.10) and taking into account that $\alpha_{pq} = \alpha_p$, $\beta_{pq} = \beta_p$, $\theta_{pq} = \theta_p$ at $z = 0$, we obtain

$$(F + \nu_1^2 C)\nu_1\omega_{p1} + (F + \nu_2^2 C)\nu_2\omega_{p2} = (F + \nu_p^2 C)\nu_p\omega_p$$
$$(F + \nu_1^2 C)\omega_{p1} + (F + \nu_2^2 C)\omega_{p2} = -(F + \nu_p^2 C)\omega_p \tag{3.6}$$

from which

$$\omega_{pq} = \frac{F + \nu_p^2 C}{F + \nu_q^2 C}\,\frac{\nu_p + \nu_{q1}}{\nu_p - \nu_{q1}}\,\omega_p = iA_{pq}D_p, \qquad q_1 = 3 - q \tag{3.7}$$

Therefore, we have for the radial displacement and the displacement in the z-direction

$$u_{\rho pq}^\circ = \frac{(L - \nu_q^2 C)\,A_{pq}D_p\,(\nu_p z_0 + \nu_q z)}{\rho R_{pq}}, \qquad w_{pq} = \frac{-(L + F)\,A_{pq}\nu_q D_p}{R_{pq}} \tag{3.8}$$

The functions $u_{\rho pq}^\circ$ are unbounded for $\rho = 0$. Hence, we transform from the solution (3.8) to the new pq-solution obtained from (3.8) by substituting the expression $1 - (\nu_p z_0 + \nu_q z)/R_{pq}$ for the fraction $\nu_p z_0 + \nu_q z\,/\,R_{pq}$ in the formula for $u_{\rho pq}^\circ$, which is equivalent to adding to (3.8) a particular solution of the form

$$u^\circ = A^\circ\xi\rho^{-2}, \qquad v^\circ = A^\circ\eta\rho^{-2} \qquad w^\circ = 0$$
$$A^\circ = \text{const} \tag{3.9}$$

which satisfies the boundary conditions and the conditions at infinity. The need to add it can be discarded for other boundary conditions, say, rigid framing of the half-space boundary. We obtain

$$u_{\rho pq} = \frac{(L - \nu_q^2 C)\,A_{pq}D_p}{R_{pq}R_{pq}^*}, \qquad w_{pq} = -\frac{(L + F)\,A_{pq}\nu_q D_p}{R_{pq}}$$
$$R_{pq}^* = R_{pq} + \nu_p z_0 + \nu_q z \tag{3.10}$$

The final formulas for elastic displacements are written as

$$u_\rho = -\frac{(L+F)E_0}{(\nu_1-\nu_2)^2}\rho\sum_{p=1}^{2}\left[\frac{\nu_p-\nu_{p1}}{R_pR_p^*}-\frac{\nu_p+\nu_{p1}}{R_{pp}R_{pp}^*}+\frac{F+\nu_{p1}^2C}{F+\nu_p^2C}\frac{2\nu_p}{R_{pp_1}R_{pp_1}^*}\right]$$
$$w = \frac{-E_0}{(\nu_1-\nu_2)^2}\sum_{p=1}^{2}(L-\nu_{p1}^2C)\nu_p\left[\frac{\nu_p-\nu_{p1}}{R_p}-\frac{\nu_p+\nu_{p1}}{R_{pp}}+\frac{F+\nu_p^2C}{F+\nu_{p1}^2C}\frac{2\nu_{p1}}{R_{pp1}}\right] \quad (3.11)$$
$$(p_1 = 3-p)$$

For $z_0 = 0$ we derive a solution from (3.11) which corresponds to the effect of a concentrated force on a half-space boundary [6] (*)

$$u_\rho = \frac{P(L+F)\rho\nu_1\nu_2}{2\pi L(\nu_2-\nu_1)}\left[\frac{\nu_1}{A+\nu_1^2F}\frac{1}{R_1R_1^*}-\frac{\nu_2}{A+\nu_2^2F}\frac{1}{R_2R_2^*}\right]$$
$$w = \frac{P(L+F)\nu_1\nu_2}{2\pi L(\nu_2-\nu_1)}\left[\frac{\nu_2^2}{A+\nu_2^2F}\frac{1}{R_1}-\frac{\nu_1^2}{A+\nu_1^2F}\frac{1}{R_2}\right] \quad (3.12)$$

Putting

$$A = C = \lambda + 2\mu, \qquad F = \lambda, \qquad L = \mu$$

in (3.11), and resolving the indeterminacy, we obtain the known Mindlin solution [8].

4. Concentrated force at a point of composite space. Here, besides the "reflected" $p\dot{q}$-solution it is necessary to take account of the "refracted" pq_1-solution connected with the variable $\theta_{pq}^{(1)}$ defined by the relationship

$$\delta_{pq}^{(1)} = \alpha_{pq}^{(1)}\xi + \beta_{pq}^{(1)}\eta - i\nu_p z_0 + i\nu_q^{(1)}z = 0 \quad (4.1)$$

Particular solutions of the form

$$u_p^* = u_p + u_{p1}^\circ + u_{p2}^\circ + u_{p1}^{(1)} + u_{p2}^{(1)}, \ldots \quad (4.2)$$

are selected in such a manner that given conditions of coupling the considered transversally isotropic half-spaces would be satisfied. For example, in the case of a smooth contact, we have at the interface $z = 0$

$$\sigma_z = \sigma_z^{(1)}, \qquad w = w^{(1)}, \qquad \tau_{z\rho} = \tau_{z\rho}^{(1)} = 0 \quad (4.3)$$

If a concentrated force of intensity P acts at the point M_0 (x_0, y_0, z_0) of a half-space with the elastic constants $A, C, F, \ldots$, then in the absence of torsion we have in place of (3.6)

$$L(F+\nu_1^2C)\omega_{p1} + L(F+\nu_2^2C)\omega_{p2} - L^{(1)}(F^{(1)}+\nu_1^{(1)2}C^{(1)})\omega_{p1}^{(1)} - L^{(1)}(F^{(1)}+\nu^{(1)2}C^{(1)})\omega^{(1)}{}_{p2} = -L(F+\nu_p^2C)\omega_p$$
$$\nu_1(F+\nu_1^2C)\omega_{p1} + \nu_2(F+\nu_2^2C)\omega_{p2} = \nu_p(F+\nu_p^2C)\omega_p$$
$$\nu_1^{(1)}(F^{(1)}+\nu_1^{(1)2}C)\omega_{p1}^{(1)} + \nu_2^{(1)}(F^{(1)2}+\nu_2^{(1)2}C)\omega_{p2}^{(1)} = 0 \quad (4.4)$$
$$\nu_1\omega_{p1} + \nu_2\omega_{p2} + \nu_1^{(1)}\omega_{p1}^{(1)} + \nu_2^{(1)}\omega_{p2}^{(1)} = \nu_p\omega_p$$

The relationships (4.4) allow the construction of a solution of the formulated problem in elementary form, and in particular, obtaining the appropriate solution for an isotropic composite space comprised of isotropic half-spaces of various materials. However, because of the awkwardness of the formulas obtained, it will be presented just for the case when the materials of the half-spaces are identical $A^{(1)} = A, \ldots$

Under the mentioned conditions we have

*) It is necessary to eliminate the inaccuracy in evaluating the integrals (4.12) in [6] p. 1103.

$$\omega_{pq} = {}^1/_2 (S_{pq} + Q_{pq}), \quad \omega_{pq}^{(1)} = {}^1/_2 (S_{pq} - Q_{pq})$$

$$S_{pq} = \frac{\nu_p(\nu_{q1} - \nu_p)}{\nu_q(\nu_{q1} - \nu_q)}, \qquad Q_{pq} = -\frac{F + \nu_p^2 C}{F + \nu_q^2 C} \frac{\nu_p + \nu_{q1}}{\nu_{q1} - \nu_q} \tag{4.5}$$

Substituting (4.5) into formulas for displacements of the form (1.12) and adding the solution of the form (3.9), we find the desired solution

$$u_\rho = -\frac{E_0(L+F)}{(\nu_1 - \nu_2)^2} \rho \sum_{p=1}^{2} \left[\frac{\nu_p - 2\nu_{p1}}{R_p R_p^*} - \frac{\nu_p}{R_{pp} R_{pp}^*} + \right.$$
$$\left. + \frac{F + \nu_{p1}^2 C}{F + \nu_p^2 C}\left(\frac{\nu_p}{R_{pp_1} R_{pp_1}^*} + \frac{\nu_p}{R_{pp_1}^{(1)} R_{pp_1}^{(1)*}}\right)\right]$$

$$w = -\frac{E_0}{(\nu_1 - \nu_2)^2} \sum_{p=1}^{2} (L - \nu_{p_1}^2 C)\nu_p \left[\frac{\nu_p - 2\nu_{p_1}}{R_p} - \frac{\nu_p}{R_{pp}} + \right.$$
$$\left. + \frac{F + \nu_p^2 C}{F + \nu_{p_1}^2 C}\left(\frac{\nu_{p_1}}{R_{pp_1}} + \frac{\nu_{p_1}}{R_{pp_1}^{(1)}}\right)\right] \tag{4.6}$$

$$R_{pp_1} = [\rho^2 + (\nu_{p_1} z - \nu_p z_0)^2]^{1/2}, \quad R_{pp_1}^{(1)*} = R_{pp_1}^{(1)} + \nu_{p_1} z - \nu_p z_0$$

By a passage to the limit, the appropriate solution can be derived for an isotropic medium from (4.6).

For $z_0 = 0$ we again obtain (3.12), which is suitable to describe the state of stress of both half-spaces.

It is easy to examine the case when the interface between the half-spaces is not perpendicular to the axis of elastic symmetry. For example, if the plane $y = 0$ is the interface, then it is convenient to represent the solution (2.4) in the form

$$u = -\mathrm{Re}[(L - \nu_1^2 C)\alpha_1(\delta_1')^{-1}\omega_1 + (L - \nu_2^2 C)d_2(\delta_2^1)^{-1}\omega_2]$$
$$v = -\mathrm{Re}[(L - \nu_1^2 C)\beta_1(\delta_1')^{-1}w_1 + (L - \nu_2^2 C)\beta_2(\delta_2')^{-1}\omega_2]$$
$$w = (L + F)\mathrm{Re}[(\delta_1')^{-1}\omega_1 + (\delta_2')^{-1}\omega_2] \tag{4.7}$$

Here

$$\delta_p = \alpha_p\xi + \beta_p\eta + \zeta = 0, \quad \delta_p' = \alpha_p'\xi + \beta_p'\eta \quad (p = 1, 2)$$
$$\alpha_p' = d\alpha_p / d\theta_p, \quad \beta_p' = d\beta_p / d\theta_p, \quad \alpha_p = \theta_p$$
$$\beta_p = i\sqrt{\nu_p^{-2} + \theta_p^2}, \quad \omega_p = D_p\beta_p^{-1} \tag{4.8}$$

The "reflected" and "refracted" solutions are connected with variables defined by the relationships

$$\delta_{pq} = \alpha_{pq}\xi - \beta_{pq}y - \beta_p y_0 + \zeta = 0 \qquad \begin{pmatrix} p = 1, 2 \\ q = 1, 2, 3 \end{pmatrix}$$
$$\delta_{pq}^{(1)} = \alpha_{pq}^{(1)}\xi + \beta_{pq}y - \beta_p y_0 + \zeta = 0$$
$$\alpha_{pq} = \theta_{pq}, \quad \beta_{pq} = i\sqrt{\nu_q^{-2} + \theta_{pq}^2}$$
$$\alpha_{pq} = \theta_{pq}^{(1)}, \quad \beta_{pq}^{(1)} = i\sqrt{\nu_q^{(1)-2} + \theta_{pq}^{(1)2}} \tag{4.9}$$

All the variables coincide at the interface. The solutions metioned above, with subscript pq, have been chosen from the accepted coupling conditions. The scheme of the solution remains as before.

Let us note that the solution does not turn out to be elementary for $y_0 \neq 0$, and requires solution of a fourth power algebraic equation. However, for $y_0 = 0$, i.e. under the effect of a concentrated force along the interface, it again becomes elementary.

In conclusion, let us note the possibility of applying the method to solve the same problems for media with a more general kind of anisotropy.

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STRESS CONDITIONS IN PLATES REINFORCED BY STIFFENING RIBS

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The problem of sresses transmitted through a stiffening rib in a plate is usually examined under various simplifying assumptions (see e. g. [1–5]).

A sufficiently simple method is proposed below for effective construction of solutions for problems of this type. This approach based on known methods of solution of planar problems permits to construct the solution in finite form.

The solution is found in integrals of the Cauchy type. The density of these integrals is determined by means of Fourier transformation.

1. The method of solution will be presented using as an example an elastic half-plane reinforced by a semi-infinite straight stringer (stiffening rib) continuously attached to the plane along the boundary.

We shall assume that the stresses (in the plate and in the stringer) are produced by only one axial force applied at the end of the stringer.

We locate the plate in the lower half-plane of the plane of the complex variable $z = x + iy$ and let the stringer coincide with the positive part of the real axis. One end